Dynamics of an active magnetic particle in a rotating magnetic field

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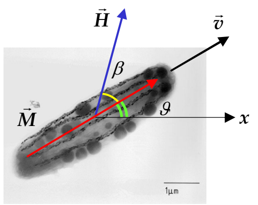
1. Introduction

In nature there are numerous examples of organisms which are sensitive to magnetic field (see [1] for overview). The best known example is pigeon, but there are also smaller ones like magnetotactic bacteria, which are able to produce the so called magnetosomes inside their bodies. They are so small that magnetic field can literally steer them. In fact the survival of magnetotactic bacteria depends a lot on magnetic field because if they are taken to opposite hemisphere of Earth, they will almost definitely die.

The study of the behaviour of magnetic particles in a rotating magnetic field has a rather long history. However self-propelled magnetic particles have not been properly investigated. In [3] we provide a simple model for such particles and here we demonstrate the diversity of possible trajectories and analyse them.

2. Model

We consider an active self-propelled magnetic particle in two dimensions. It moves with constant velocity v in the direction of its magnetic moment M. By applying external magnetic field H, we can influence the direction of motion.



We fix an arbitrary direction to be x axis and introduce the following angles: ϑ - orientation angle, β – the angle between external field H and magnetic moment M. Then coordinates of particle are changing according to

$$\frac{dx}{dt} = v\cos\vartheta \qquad \frac{dy}{dt} = v\sin\vartheta$$

and torque balance for particle reads

$$-\alpha\frac{d\vartheta}{dt} + MH\sin\beta = 0$$

where α is the rotational friction coefficient. For rotating magnetic field we have $\beta = \omega t - \vartheta$ and torque balance is

$$\frac{d\beta}{dt} = \omega - \omega_c \sin\beta$$

where $\omega_c = MH/\alpha$ is critical frequency.

3. Solution

If $\omega \le \omega_c$, there is only a stationary solution $\sin\beta_0 = \omega/\omega_c$ which corresponds to motion around circle.

If $\omega > \omega_c$ particle cannot rotate synchronously with the applied field and angle β is a periodic function of time:

$$\beta = 2\arctan\left(\gamma + \sqrt{1-\gamma^2}\tan\frac{\sqrt{1-\gamma^2}\,\omega(t-t_0)}{2}\right)$$

where $\gamma = \omega/\omega_c < 1$ and time moment t_0 can be chosen arbitrarily (we choose it equal to zero).

4. Analysis

4.1. Commensurability of periods

The shape of trajectory depends on commensurability of periods of particle orientation and field rotation. Period of orientational motion of particle is given by

$$T_1 = T/\sqrt{1-\gamma^2}$$

where $T = 2\pi/\omega$ is the field period. The condition of periodicity is that quantity

$$\frac{T_1}{T} = \sqrt{1-\gamma^2}$$

is rational. If periods are commensurable, the trajectory is closed. Otherwise it covers the accessible region densely.

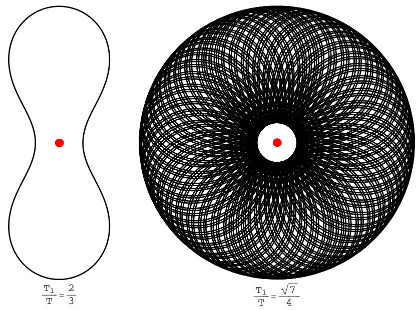


Figure 1 Trajectories for two close rational and irrational values of ratio T_1/T.

4.2. Winding number

Winding number Wn describes the change of particles orientation during one period of its orientational motion (when deviation angle β ranges over $[0, 2\pi]$)

$$Wn = \frac{\Delta\vartheta}{2\pi} = \int_0^{2\pi}\frac{\sin\beta}{1-\gamma\sin\beta}d\beta = \frac{1}{\sqrt{1-\gamma^2}} - 1$$

The integer part of Wn determines the number of full turns made at each small circle. If Wn is integer, then the trajectory of particle in average is a straight line.

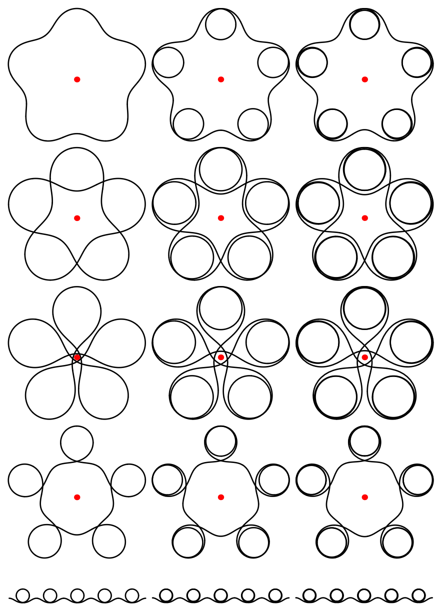


Figure 2 Shape of trajectory for different rational values of winding number. In all cases the denominator is 15, but numerator increases from top to bottom and from left to right. In each successive column the number of full turns at small circles is increased by one.

4.3. Curvature

The dimensionless curvature k of trajectory is

$$k = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\sqrt{(\dot{x}^2+\dot{y}^2)^3}} = \sin\beta$$

It is very typical that k almost equals 1 for relatively long time. These intervals are separated by peaks with negative curvature.

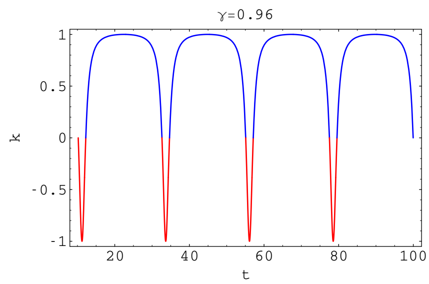


Figure 3 Typical plot of dimensionless curvature k.

At those points where curvature equals zero, the direction of motion coincides with the direction of external field. At small circles the magnetic force acts as a centripetal force.

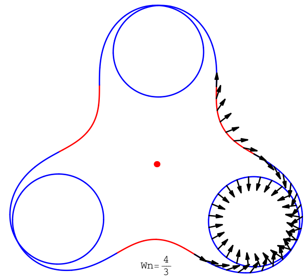


Figure 4 Trajectory of particle where blue segments denote positive curvature and red segments – negative. Arrows indicate the direction of external field when particle passes corresponding point.

4.4. Direction of rotation

Resulting circulation is possible in both directions: clockwise and anticlockwise. There are such values of γ when the direction of resulting circulation is extremely sensitive.

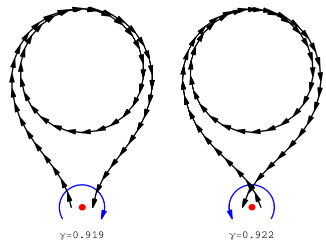


Figure 5 Transition from one direction of rotation to other.

It happens when particle passes through the geometric centre of its trajectory.

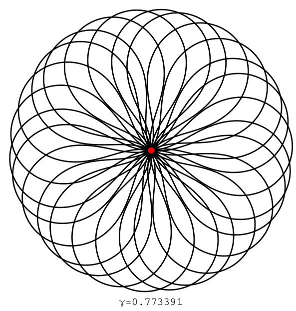


Figure 6 Trajectory with undetermined direction of circulation (particle goes through the centre).

5. Conclusion

We provide a mathematical model for self-propelled magnetic particle in a rotating magnetic field. It can be used to describe the motion of magnetotactic bacteria. Our model predicts two possible regimes (at high and low field frequencies), which both were experimentally observed in [2]. We also provide elaborate analysis of possible trajectories. They fit well with experimentally obtained [2].

Our model can be used to calculate some physical quantities, which are almost impossible to obtain by direct measurement, for example rotational friction coefficient of bacteria. Yet it can be also used to generate beautiful symmetric patterns.

6. References

1. M.Winklhofer, Biogenic magnetite and magnetic sensitivity in organisms - from magnetic bacteria to pigeons, Magnetohydrodynamics Vol. 41 (2005), No. 4, pp. 295-304.
2. B.Steinberger, N.Petersen, H.Petermann, D.G.Weiss, Movement of magnetic bacteria in time-varying magnetic fields, J.Fluid.Mech. (1994), Vol. 273, pp. 189-211.
3. A.Cēbers, M.Ozols, Dynamics of an active magnetic particle in a rotating magnetic field, Phys.Rev.E 73, 021505 (2006).

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